Is there a Golden Parachute in Sannikov's principal-agent problem ?

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Agent and Principal Criteria

Agent is in charge of the output process

$$dX_t \quad = \quad \alpha_t dt + \sigma dW_t, \quad \alpha_t \in A \quad (\text{compact } \subset \mathbb{R}_+,\ 0 \in A)$$

and chooses effort process α so as to balance his cost of effort $h(\alpha_s)$ by maximizing the utility criterion :

$$J_0^{\mathrm{A}}(\pi, \alpha) := \mathbb{E}^{\mathbb{P}^\alpha}\left[\int_0^\infty r e^{-rs}\big(u(\pi_s) - h(\alpha_s)\big)\mathrm{d}s\right]$$

Given Agent's optimal response $\hat{\alpha}(\pi)$, Principal chooses best compensation scheme so as to maximize

$$J_0^{\mathrm{P}}(\pi, \hat{\alpha}(\pi)) := \mathbb{E}^{\mathbb{P}^{\hat{\alpha}(\pi)}}\left[\int_0^\infty r e^{-rt}(\mathrm{d}X_t - \pi_t \mathrm{d}t)\right] = \mathbb{E}^{\mathbb{P}^{\hat{\alpha}}}\left[\int_0^\infty r e^{-rt}(\hat{\alpha}_t - \pi_t)\mathrm{d}t\right]$$

under Agent's participation constraint

$$J_0^{\mathrm{A}}(\pi, \hat{\alpha}(\pi)) \geq R$$

Continuation utility and Golden Parachute

Motivated by previous discrete time literature, Sannikov guesses that the continuation utility of the principal V_t^{P} only depends on the continuation utility Y_t of the agent

$$V_t^{\mathrm{P}} = V(Y_t),$$

Remarks : 1) $Y_t \geq 0$ because $u(.) \geq 0$ and zero effort is legitimate

2) Golden Parachute : constant lifetime payment π_τ on $[\tau, \infty)$. Then

$$Y_t = u(\pi_\tau) \quad \text{and} \quad V_t^{\mathrm{P}} = V(Y_t) = -\pi_\tau = -u^{-1}(Y_t) \;\; \text{for all} \;\; t \geq \tau$$

Definition

The model exhibits a Golden Parachute if the principal value function satisfies $V = F := -u^{-1}$ on $[y_{\mathrm{gp}}, \infty)$ for some $y_{\mathrm{gp}} > 0$

We write **NGP** for No Golden Parachute

NGP in the first–best version of Sannikov's problem

The first–best optimal contracting problem is :

$$V_0^{\mathrm{P,FB}} := \sup\Big\{ J_0^{\mathrm{P}}(\pi,\alpha) : \pi \in \Pi, \alpha \in \mathcal{A}, \text{ and } J_0^{\mathrm{A}}(\pi,\alpha) \geq R \Big\}$$

Let $G^\star(p) := \sup\limits_{a\geq 0}\big\{a + ph(a)\big\}$, and $F^\star(p) := \inf_{a\geq 0}\big\{a+pu(a)\big\}$

Proposition

$V_0^{\mathrm{P,FB}} = \psi(R) := \inf_{p\geq 0}\big\{Rp - (F^\star - G^\star)(p)\big\}$, with optimal contract

$$u(\pi^\star) = h(a^\star) = (F^\star)'(\lambda^\star), \text{ where } \lambda^\star = \psi'(R).$$

In particular, optimal contract binds the participation constraint.

By standard Kuhn–Tucker method,

$$\begin{aligned}
&\inf_{\lambda\geq 0}\Big\{\lambda R + \sup_{\pi,\alpha}\mathbb{E}^{\mathbb{P}^\alpha}\Big[\int_0^\infty re^{-rt}\Big(-\big(\pi_t+\lambda u(\pi_t)\big) + \big(\alpha_t + \lambda h(\alpha_t)\big)\Big)\mathrm{d}t\Big]\Big\}\\
&= \inf_{\lambda\geq 0}\Big\{\lambda R + \sup_{\pi,\alpha}\mathbb{E}^{\mathbb{P}^\alpha}\Big[\int_0^\tau re^{-rt}\Big(-F^\star(\lambda) + G^\star(\lambda)\Big)\mathrm{d}t\Big]\Big\}\\
&= \inf_{\lambda\geq 0}\big\{\lambda R - (F^\star - G^\star)(\lambda)\big\} = \psi(R)
\end{aligned}$$

Dynamics of Agent's continuation utility

By standard dynamic programming (Martingale optimality principle)

$$dY_t = Z_t dX_t + \big[rY_t dt + ru(\pi_t) + H(Z_t)\big] dt \text{ where } H(z) := \sup_a \{az - rh(a)\}$$

with optimal effort process satisfying $\hat{a}_t \in \hat{A}(Z_t) := \mathrm{Argmax}\, H(Z_t)$

Sannikov's elegant method

By standard dynamic programming (Martingale optimality principle)

$$dY_t = r\big(Z_t dX_t + \big[Y_t dt + u(\pi_t) + H(Z_t)\big]dt\big) \text{ where } H(z) := \sup_a \big\{az - h(a)\big\}$$

with optimal effort process satisfying $\hat{a}_t \in \hat{A}(Z_t) := \mathrm{Argmax} H(Z_t)$

• This trick turns out to be extremely efficient, and was used by many authors since then

• This was the starting point of Cvitanić, Possamaï & NT '17, extended to random horizon by Lin, Ren, NT & Yang '20.

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Reduction to a control problem

Then, Principal's problem reformulated as

$$V^{\mathrm{P}} = \sup_{\pi, Z} \sup_{\hat{\alpha} \in \hat{A}(Z)} \mathbb{E}^{\mathbb{P}^{\hat{\alpha}}} \left[\int_0^\infty r e^{-rt} (\hat{\alpha}_t - \pi_t) \mathrm{d}t \right]$$

where $dY_t = r(Y_t - u(\pi_t) + h(\hat{\alpha}_t))dt + rZ_t dW_t^{\hat{\alpha}}$

Sannikov guesses that V^{P} is expected to satisfy the HJB equation

$$v - yv' - \sup_{\pi} \left\{ -\pi + u(\pi) v' \right\} - \sup_{z, \hat{a} \in \hat{A}(z)} \left\{ \hat{a} + h(\hat{a}) v' + \frac{1}{2} r \sigma^2 z^2 v'' \right\} = 0 \text{ on } [0, y_{\mathrm{gp}}]$$

with $v(0) = 0$, where y_{gp} is a free boundary such that

$$V(y_{\mathrm{gp}}) = F(y_{\mathrm{gp}}) \quad \text{and} \quad V'(y_{\mathrm{gp}}) = F'(y_{\mathrm{gp}}) \quad \text{(smoothfit)}$$

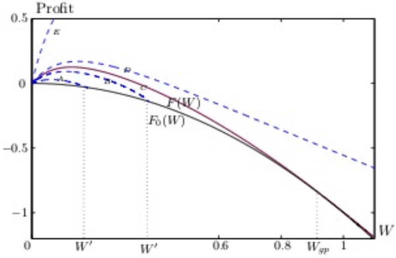
and proceeds by verification

Sannikov's main findings

Assume
- $u \in C^1$, strictly concave increasing, $u(0) = 0$, $u'(\infty) = 0$
- $h \in C^1$, increasing and strictly convex, and $\beta := h'(0) > 0$

Sannikov (RES '08)

(i) There exists a solution v to the HJB equation
(ii) $V^{\mathrm{P}} = \sup_{Y_0 \geq R} v(Y_0)$
(iii) For small $Y_0 > 0$, we have $V^{\mathrm{P}} = v(Y_0^*)$ for some $Y_0^* > Y_0$ (informational rent)



Objectives of our work

- Rigorous formulation of the continuous time contracting problem
- Golden Parachute is not always optimal !
- Informational rent near the origin is always true
- All results hold when Agent and Principal have different discount factors
  - requires to introduce a face-lifted agent's utility function
  - contrasting some conjectures / claims by Sannikov

Contracts

$\mathbf{C} := (\tau, \pi, \xi)$: τ stopping time : retirement time
π : non–negative continuous payment
$\xi \in \mathbb{L}^0(\mathcal{F}_\tau)$: non–negative payment at retirement ($\sim$ GP)

satisfying for some $r' \in (0, r \wedge \frac{\rho}{\gamma})$:

$$\lim_{n\to\infty} \sup_{\alpha\in\mathcal{A}} \mathbb{P}^\alpha[\tau \geq n] = 0, \text{ and } \sup_{\alpha\in\mathcal{A}} \mathbb{E}^{\mathbb{P}^\alpha}\left[\left(\mathrm{e}^{-r'\tau}|\xi|\right)^\gamma + \int_0^\tau \left(\mathrm{e}^{-r's}|\pi_s|\right)^\gamma \mathrm{d}s\right] < \infty$$

- r, ρ discount factors of Agent and Principal
- $\gamma > 1$: parameter related to the agent's utility function

$$c_0\left(-1 + \pi^{\frac{1}{\gamma}}\right) \leq u(\pi) \leq c_1\left(1 + \pi^{\frac{1}{\gamma}}\right), \quad \pi \geq 0, \quad \text{for some} \quad c_0, c_1 \geq 0$$

The contracting problem

Utility criteria of Agent and Principal :

$$J^{\rm A}(\mathbf{C},\alpha) := \mathbb{E}^{\mathbb{P}^\alpha}\Big[\mathrm{e}^{-r\tau}u(\xi) + \int_0^\tau r\mathrm{e}^{-rs}\big(u(\pi_s) - h(\alpha_s)\big)\mathrm{d}s\Big]$$

$$J^{\rm P}(\mathbf{C},\alpha) := \mathbb{E}^{\mathbb{P}^\alpha}\Big[-\mathrm{e}^{-\rho\tau}\xi + \int_0^\tau \rho\mathrm{e}^{-\rho s}(-\pi_s + \alpha_s)\mathrm{d}s\Big]$$

Agent's utility maximization problem :

$$V^{\rm A}(\mathbf{C}) := \sup_{\alpha\in\mathcal{A}} J^{\rm A}(\mathbf{C},\alpha), \text{ and } \widehat{\mathcal{A}}(\mathbf{C}) := \big\{\widehat{\alpha}\in\mathcal{A} : V^{\rm A}(\mathbf{C}) = J^{\rm A}(\mathbf{C},\widehat{\alpha})\big\}$$

Principal's problem :

$$V^{\rm P} := \sup_{\mathbf{C}\in\mathfrak{C}_R}\ \sup_{\widehat{\alpha}\in\widehat{\mathcal{A}}(\mathbf{C})} J^{\rm P}(\mathbf{C},\alpha), \quad \text{where} \quad \mathfrak{C}_R := \big\{\mathbf{C}\in\mathfrak{C} : V^{\rm A}(\mathbf{C}) \geq R\big\}$$

The contracting problem

Utility criteria of Agent and Principal : $F := -u^{-1}\mathbf{1}_{[0,u(\infty))} - \infty\mathbf{1}_{[0,u(\infty))^c}$

$$J^{\mathrm{A}}(\mathbf{C},\alpha) := \mathbb{E}^{\mathbb{P}^\alpha}\left[\mathrm{e}^{-r\tau}\zeta + \int_0^\tau r\mathrm{e}^{-rs}\big(\eta_s - h(\alpha_s)\big)\mathrm{d}s\right]$$

$$J^{\mathrm{P}}(\mathbf{C},\alpha) := \mathbb{E}^{\mathbb{P}^\alpha}\left[\mathrm{e}^{-\rho\tau}F(\zeta) + \int_0^\tau \rho\mathrm{e}^{-\rho s}(F(\eta_s) + \alpha_s)\mathrm{d}s\right]$$

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$$V^{\mathrm{A}}(\mathbf{C}) := \sup_{\alpha\in\mathcal{A}} J^{\mathrm{A}}(\mathbf{C},\alpha), \text{ and } \widehat{\mathcal{A}}(\mathbf{C}) := \{\widehat{\alpha}\in\mathcal{A} : V^{\mathrm{A}}(\mathbf{C}) = J^{\mathrm{A}}(\mathbf{C},\widehat{\alpha})\}$$

Principal's problem :

$$V^{\mathrm{P}} := \sup_{\mathbf{C}\in\mathfrak{C}_R}\ \sup_{\widehat{\alpha}\in\widehat{\mathcal{A}}(\mathbf{C})} J^{\mathrm{P}}(\mathbf{C},\alpha), \quad \text{where} \quad \mathfrak{C}_R := \{\mathbf{C}\in\mathfrak{C} : V^{\mathrm{A}}(\mathbf{C}) \geq R\}$$

Face-lifted utility

Principal's relaxed problem : $V^{\mathrm{P}} = \sup_{\mathbf{C}\in\mathfrak{C}_R} \sup_{\alpha\in\mathcal{A}^\star(\mathbf{C})} \bar{J}^{\mathrm{P}}(\mathbf{C},\alpha)$,

$$\bar{J}^{\mathrm{P}}(\mathbf{C},\alpha) := \mathbb{E}^{\mathbb{P}^\alpha}\Big[\mathrm{e}^{-\rho\tau}\overline{F}(\zeta) + \int_0^\tau \rho\mathrm{e}^{-\rho t}\big(\alpha_t + F(\eta_t)\big)\mathrm{d}t\Big].$$

with **face-lifted** (inverse) utility

$$\overline{F}(y_0) \quad := \quad \sup_{p(.)\geq 0} \; \sup_{T\in[0,T_0^{y_0,p}]} \Big\{\mathrm{e}^{-\rho T} F\big(y^{y_0,p}(T)\big) + \int_0^T \rho\mathrm{e}^{-\rho t} F\big(p(t)\big)\mathrm{d}t\Big\}$$

where $T_0^{y_0,p} := \inf\big\{t \geq 0 : y^{y_0,p}(t) \leq 0\big\}$, and

$$y^{y_0,p}(0) = y_0, \; \dot{y}^{y_0,p}(t) = r\big(y^{y_0,p}(t) - p(t)\big), \; t > 0$$

Proof : as $y_0 = \mathrm{e}^{-rT}y(T) + \int_0^T \mathrm{e}^{-rt}p(t)\mathrm{d}t$, Agent is indifferent between

- lump–sum payment $\xi(\omega)$ at $\tau(\omega)$,
- payment $p(t)$ and zero effort on $[\tau(\omega), \tau(\omega) + T]$, and retirement deferred to $\tau(\omega) + T$, with the lump–sum payment $\xi'(\omega) := u\big(y^{\zeta(\omega),p}(T)\big)$

Face-lifted utility in explicit form

Introduce the concave conjugate functions

$$F^\star(p) := \inf_{y\geq 0}\big\{yp - F(y)\big\}, \text{ and } \overline{F}^\star(p) := \inf_{y\geq 0}\big\{yp - \overline{F}(y)\big\},\ p\in\mathbb{R}.$$

Proposition

Assume that $\gamma\delta > 1$. Then, $\overline{F} = (\overline{F}^\star)^\star$ is decreasing, strictly concave,

- $\overline{F} = F$ for $\delta = 1$
- $\overline{F}^\star(p) = \frac{1}{1-\delta}\big(\frac{|p|}{\delta}\big)^{\frac{1}{1-\delta}} \int_{b_\delta}^{\delta p} |x|^{-1-\frac{1}{1-\delta}} F^\star(x)\mathrm{d}x$, with $b_\delta := -\infty$ for $\delta < 1$, and $b_\delta := 0$ for $\delta > 1$

Moreover, $\overline{F}(y_0) = \sup_{p\geq 0} \int_0^{T^{y_0,p}} \rho e^{-\rho t} F\big(p(t)\big)\mathrm{d}t,\ y_0 \geq 0$

Golden Parachute

Definition

We say that the contracting model exhibits a **Golden Parachute**, if there exists an optimal contract $(\tau^\star, \pi^\star, \xi^\star) \in \mathfrak{C}_R$ for the relaxed formulation of the principal problem such that $\tau^\star > 0$, and $\mathbb{P}[\xi^\star > 0] > 0$.

In words, GP means that Agent

- ceases any effort at some positive stopping time,
- and retires with non-zero payment

Reduction to a mixed control–stopping problem

Agent's Hamiltonian

$$h^\star(z) := \sup_{a\in A} \{za - h(a)\}, \text{ with } \hat{A}(z) := \{a \in A : h^\star(z) = za - h(a)\}$$

By the reduction result of Lin, Ren, NT & Yang '20, we have

$$V^{\mathrm{P}} = \sup_{Y_0 \geq R} V(Y_0), \text{ where } V(Y_0) := \sup_{(\tau,Z,\pi)\in\mathcal{Z}(Y_0)} \sup_{\hat{a}\in\hat{A}(Z)} \overline{J}(\tau,\pi,Z,\hat{a})$$

and

$$\overline{J}(\tau,\pi,Z,\hat{a}) := \mathbb{E}^{\mathbb{P}^{\hat{a}}}\left[\int_0^\tau \rho e^{-\rho t}\big(\hat{a}_t + F(\eta_t)\big)dt + e^{-\rho\tau}\overline{F}\big(Y_\tau^{Y_0,Z,\pi}\big)\right]$$

with controlled state

$$dY_t^{Y_0,Z,\pi} = r\big(Y_t^{Y_0,Z,\pi} + h(\hat{a}_t) - \eta_t\big)dt + rZ_t\sigma dW_t^{\hat{a}}$$

Reduction to a mixed control–stopping problem

Define the parameters $\delta := \frac{r}{\rho}$ and $\eta := \frac{1}{2} r\sigma^2$, the dynamic programming equation is :

$$\text{(DPE)} \quad v(0) = 0, \text{ and } \min\left\{v - \overline{F}, \mathbf{L}v\right\} = 0, \text{ on } (0,\infty),$$

where

$$\mathbf{L}v := v - \delta y v' + F^\star(\delta v') - I\big(\delta v', \delta v''\big)$$

and

$$\begin{array}{rcl} I(p,q) & := & \infty \mathbf{1}_{\{q>0\}} + \mathbf{1}_{\{q\le 0\}} I_0(p,q)^+ \\ I_0(p,q) & := & \sup\limits_{z \ge h'(0),\ \hat{a} \in \hat{A}(z)} \left\{\hat{a} + h(\hat{a})p + \eta z^2 q\right\}. \end{array}$$

Remark : (DPE) is equivalent to $v(0) = 0$, and $\mathbf{L}v = 0$ on $(0,\infty)$

Some cases of NGP

proposition

Let $\beta := h'(0)$. Then there is No Golden Parachute whenever either

(NGP1) $\beta = 0$;

(NGP2) or $\beta > 0$, $\overline{F}''$ is non–increasing, and $I_0\big(\delta\overline{F}'(0), \delta\overline{F}''(0)\big) > 0$;

(NGP3) or $\beta > 0$, A is an interval, and $h \in C^3$ with

$$\inf\Big\{\frac{\big(h'^2\big)''}{h''}\Big\} \geq \frac{1}{\eta}\sup\Big\{-\frac{\overline{F}'}{\overline{F}''}\Big\}, \text{ and } \sup\big(\overline{F}' + 2\eta h''(0)\overline{F}''\big) \leq \frac{-1}{\delta\beta}$$

If $F'(0) = 0$ and $\overline{F}''$ non–increasing, (NGP2) holds if $\beta^2 \geq \frac{\max A}{-\eta\overline{F}''(0)}$

(NGP3) also excludes GP for large β

Proof : on the stopping region $\{v = \overline{F}\}$, we must have $\mathbf{L}\overline{F} \geq 0$...

On the importance of the face-lifted utility 1

Consider the slight modification of the criteria of Agent and Principal :

$$J_1^{\mathrm{A}}(\mathbf{C},\alpha) := \mathbb{E}^{\mathbb{P}^\alpha}\left[\mathrm{e}^{-r\tau}\zeta + \int_0^\tau \not{r}\,\mathrm{e}^{-rs}\big(\eta_s - h(\alpha_s)\big)\mathrm{d}s\right]$$

$$J_1^{\mathrm{P}}(\mathbf{C},\alpha) := \mathbb{E}^{\mathbb{P}^\alpha}\left[\mathrm{e}^{-\rho\tau}F(\zeta) + \int_0^\tau \not{\rho}\,\mathrm{e}^{-\rho s}(F(\eta_s) + \alpha_s)\mathrm{d}s\right]$$

$$V_1^{\mathrm{P}} = \sup_{Y_0 \ge R} V_1(Y_0), \text{ where } V_1(Y_0) := \sup_{(\tau,Z,\pi)\in\mathcal{Z}(Y_0)} \sup_{\hat{a}\in\hat{A}(Z)} J_1(\tau,\pi,Z,\hat{a})$$

and

$$J_1(\tau,\pi,Z,\hat{a}) := \mathbb{E}^{\mathbb{P}^{\hat{a}}}\left[\int_0^\tau \mathrm{e}^{-\rho t}\big(\hat{a}_t + F(\eta_t)\big)\mathrm{d}t + \mathrm{e}^{-\rho\tau}F\big(Y_\tau^{Y_0,Z,\pi}\big)\right]$$

with controlled state

$$\mathrm{d}Y_t^{Y_0,Z,\pi} = \big(rY_t^{Y_0,Z,\pi} + h(\hat{a}_t) - \eta_t\big)\mathrm{d}t + Z_t\sigma\mathrm{d}W_t^{\hat{a}}$$

On the importance of the face-lifted utility 1

HJB equation for this problem :

$$\text{(DPE)} \quad v(0) = 0, \text{ and } \min\left\{v - F, \mathbf{L}_1 v\right\} = 0, \text{ on } (0, \infty),$$

where

$$\mathbf{L}_1 v := \rho v - ryv' + \inf_{\eta}\left\{\eta v' - F(\eta)\right\} - \sup_{z, \hat{a} \in \hat{A}(z)}\left\{\hat{a} + h(\hat{a})v' + \frac{1}{2}\sigma^2 z^2 v''\right\}$$

Then,

$$\mathbf{L}_1 F \leq \rho F - ryF' + F^{\star}(F') = (\rho - 1)F + (1 - r)yF' < (\rho - r)F, \quad y > 0$$

as F is strictly concave. Hence, For $\rho \geq r$, we get $\mathbf{L}_1 F < 0$

If one defines GP as an instance of stopping with positive reward, then the conclusion would be NGP for $\rho \geq r$!

Wellposdness of (DPE)

Assumption : $(F^\star)'$, $\overline{F}'$, and $\overline{F}''$ are uniformly continuous. Moreover, whenever $\beta := h'(0) = 0$, then $A \supset [0, \bar{a}]$ for some $\bar{a} > 0$.

Theorem (Existence)

Let $\mathcal{S} := \{v = \overline{F}\}$. Then

(i) there exists a minimal C^2 solution v of (DPE), such that v is concave, ultimately decreasing, and $0 \leq (v - \overline{F})(y) \leq Cy$, $y \geq 0$, for some $C > 0$;

(ii) we have $v'(0) \geq 0$, and when $\overline{F}'(0) = 0$,

$$v'(0) > 0 \text{ if and only if } I\big(0, \delta\overline{F}''(0)\big) > 0\,;$$

(iii) if $\beta > 0$, and in addition the maps F and I_0 are analytic, then v is strictly concave and $\mathcal{S} = \{0\} \cup [y_{\rm gp}, \infty)$ for some $y_{\rm gp} \in [0, \infty]$;

(iv) if $\beta = 0$, then $\mathcal{S} = \{0\}$.

Solving the contracting problem

Theorem (Verification)

Under the previous conditions, and when $y_{\rm gp} < \infty$, we have

$$V^{\rm P} = \sup_{y \geq R} v(y) = v(\hat{y}), \text{ for some } \hat{y} \geq R$$

Let $\hat{z}, \hat{\pi} : [0, \infty) \longrightarrow \mathbb{R}$ be maximizers of $I(\delta v', \delta v'')$ and $F^\star(\delta v')$, then

- there is a unique weak solution $\widehat{Y} := Y^{\hat{y}, \hat{z}(\widehat{Y}), \hat{\pi}(\widehat{Y})}$ to the corresponding $\rm SDE$
- the contract $\big(\hat{\tau}, \hat{\pi}(\widehat{Y}), u^{-1}(\widehat{Y}_{\hat{\tau}})\big)$ is optimal, where

$$\hat{\tau} := \inf \left\{ t \geq 0 : \widehat{Y}_t \notin (0, y_{\rm gp}) \right\}$$

On the proof of existence

We do not understand Sannikov's argument !

- Postulates existence of $y_{\rm gp} < \infty$,
  then global solution of DPE with $v'(0) = b$ exists
- Determine $y_{\rm gp}$ by using the flow continuity in b

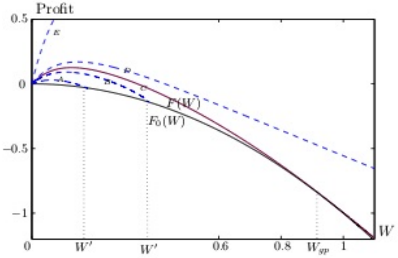

Figure 6: Typical solutions of the HJB equation.

Our argument : Prove existence of a supersolution of DPE,
then as F is a subsolution, apply Perron existence argument...

Numerical result 1

Same parameters as in Sannikov '08
$\gamma = 2$, $\eta = 0.05$, $h = 0.5$, $\beta = 0.4$, and $\delta = 1$)

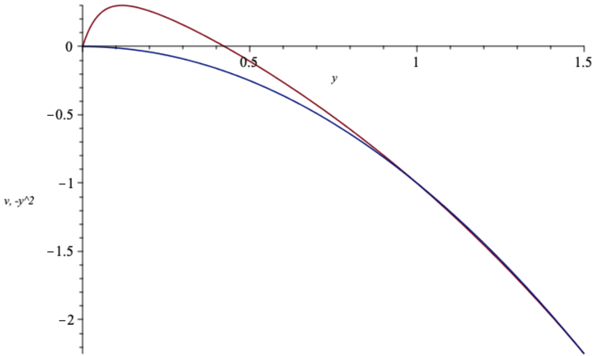

Figure – v (red), F (blue) : Golden Parachute exists

Numerical result 2

$\delta = 1$, $\gamma = 3/2$, $\eta = h = 1$, $\beta = 0.01$, and $\delta = 1$

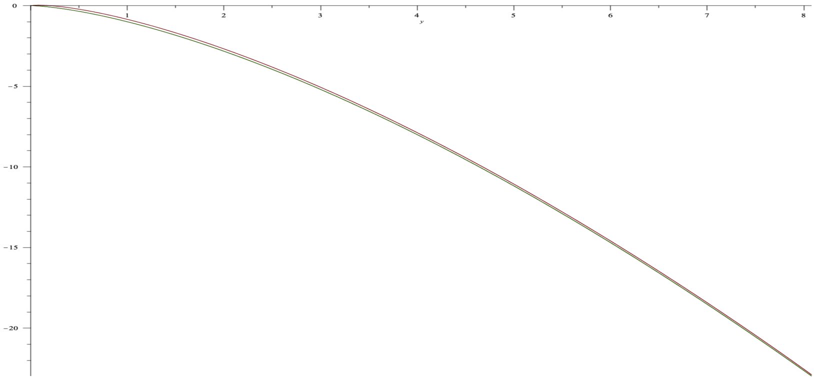

Figure – v (red), F (green) : No Golden Parachute

Numerical result 3

$\gamma = 3/2$, $\eta = h = 1$, $\beta = 0.01$, and $\delta = 3/4$
Golden parachute does exist

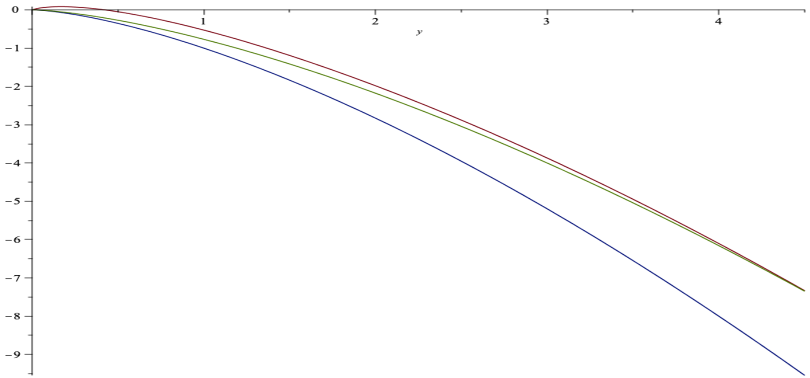

Figure – v (red), $\overline{F}$ (green), F (blue), Golden Parachute exists

Numerical result 4

$\gamma = 3$, $\eta = h = 1$, $\beta = 0.01$, and $\delta = 2$

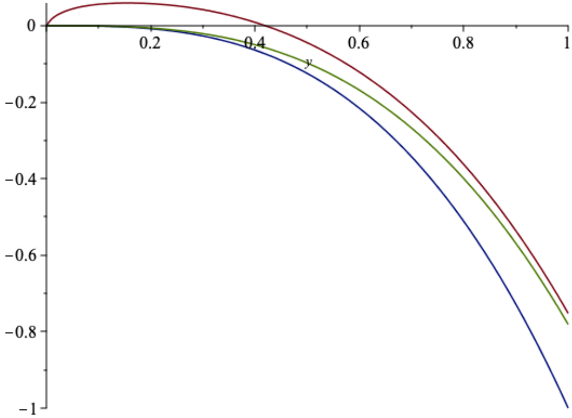

Figure – v (red), $\overline{F}$ (green), F (blue) : No Golden Parachute