Fragmenting Financial Markets

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Avoidance of price impact delays efficient trades

A workup session involving 3 buyers and 4 sellers.
Improving the initial allocation with a workup

This figure shows the inventory paths of a buyer and a seller in a market with a workup followed by a sequence of double auctions. Also shown are the inventory paths in the absence of a workup. The model is described in the next two sections. Ausubel, Cramton, Pycia, Rostek, and Weretka (2014), as well as in dynamic models of Vayanos (1999) and Du and Zhu (2014).

The two thin lines in Figure 1 illustrate the exponential convergence to the efficient allocation in double auctions, in an numerical example. We will return to this example in more detail later. We have taken the limit of $\Delta \to 0$. As shown by Vayanos (1999) and Du and Zhu (2014), even if trading is continuous, convergence to efficient allocation is still not instantaneous because dealers' strategic incentives get so strong that trading volume per round becomes essentially zero.

Now, let us add a single workup procedure before the first double auction, with the workup price taken to be the unconditional expectation of the double auction price. For tractability, we consider a restricted case in which only a single large buyer and a single large seller can potentially participate in the workup, whereas the rest of the dealers can only participate in double auctions. In the workup, the buyer and the seller continually increase the traded quantity, at the fixed price, until either side decides to exit, at which point the workup ends and the sequential double auctions.

Source: Duffie and Zhu (2017).
Size discovery in practice


▶ Scope for regulation. In 2018, European MiFiD II regulations capped dark-pool equities trade volume at 8%, and 4% for each platform.
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Main findings

1. Size discovery is highly effective for avoiding price-impact costs and can dramatically improve allocations whenever it is run.

2. The prospect of size discovery, however, reduces exchange market trade volumes and depth.

3. The net effect, as modeled, is a reduction in overall allocative efficiency.

4. Ex ante, every investor would strictly prefer the absence of size discovery.
Model Timeline

Exchange trading

0

size-discovery session

$T_1$

Exchange trading

size-discovery session

$T_2$

........
Inventory paths with and without size-discovery sessions
Static size-discovery primitives

- $n \geq 3$ traders with private initial excess inventories $z_0^1, \ldots, z_0^n$.

- Average excess inventory $\bar{Z} = (z_0^1 + \cdots + z_0^n)/n$.

- Value of excess inventory (later endogenized): 
  \[ V^i(z^i, \bar{Z}) = u^i(\bar{Z}) + \beta(\bar{Z})z^i - K \left( z^i - \bar{Z} \right)^2, \]
  where $u^i, \beta$ are functions and $K > 0$ is a scalar.

- The unique efficient allocation: $z^i = \bar{Z}$. 
Size-discovery session

- Suppose (for now) that $\bar{Z}$ is publicly observable.

- Trader $i$ submits an inventory report $\hat{z}^i$.

- Given reports $\hat{z} = (\hat{z}^1, \ldots, \hat{z}^n)$, trader $i$ gets some cash transfer $T^i(\hat{z}, \bar{Z})$ and some asset transfer $Y^i(\hat{z})$.

- Taking $\hat{z}^{-i}$ as given, trader $i$ solves

  $$\sup_{\tilde{z}} \mathbb{E}\left[V^i(z_0^i + Y^i((\tilde{z}, \hat{z}^{-i})), \bar{Z}) + T^i((\tilde{z}, \hat{z}^{-i}), \bar{Z}) \mid \mathcal{F}^i\right].$$
A linear-quadratic mechanism for size discovery

- Asset transfer

\[ Y^i(\hat{z}) = \frac{\sum_{j=1}^{n} \hat{z}^j}{n} - \hat{z}^i. \]
A linear-quadratic mechanism for size discovery

▶ Asset transfer

\[ Y^i(\hat{Z}) = \frac{\sum_{j=1}^{n} \hat{z}^j}{n} - \hat{z}^i. \]

▶ Cash transfer

\[
T^i(\hat{Z}, \bar{Z}) = \underbrace{\kappa_1(\bar{Z}) \hat{z}^i}_{\text{frozen price}} + \kappa_0 \left( n \kappa_2(\bar{Z}) + \sum_{j=1}^{n} \hat{z}^j \right)^2
\]

\[
+ \kappa_1(\bar{Z}) \kappa_2(\bar{Z}) + \frac{\kappa_1^2(\bar{Z})}{4 \kappa_0 n^2},
\]

where \( \kappa_1(\cdot) \) and \( \kappa_2(\cdot) \) are affine and \( \kappa_0 < 0 \) is a constant.
A linear-quadratic mechanism for size discovery

- Asset transfer

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T^i(\hat{z}, \bar{Z}) = \kappa_1(\bar{Z}) \hat{z}^i + \kappa_0 \left( n \kappa_2(\bar{Z}) + \sum_{j=1}^{n} \hat{z}^j \right)^2 \\
+ \kappa_1(\bar{Z}) \kappa_2(\bar{Z}) + \frac{\kappa_1^2(\bar{Z})}{4\kappa_0 n^2},
\]

where \( \kappa_1(\cdot) \) and \( \kappa_2(\cdot) \) are affine and \( \kappa_0 < 0 \) is a constant.

- Budget feasibility: \( \sum_i T^i(\hat{z}, \bar{Z}) \leq 0 \) for any \( \bar{Z} \) and \( \hat{z} \in \mathbb{R}^n \).
Proposition: For unique $\kappa_0, \kappa_1, \kappa_2$, this size-discovery mechanism is:

- **Strategy proof:** Reporting $\hat{z}_i = z_0^i$ is a strictly dominant strategy.

- **Ex-post IR:** For any $z_0 \in \mathbb{R}^n$ and for the equilibrium strategies $\hat{z}_i = z_0^i$,

$$V^i(z_0^i, \bar{Z}) \leq V^i(z_0^i + Y^i(z_0), \bar{Z}) + T^i(z_0^i, \bar{Z}).$$

- **Efficient:** $z_0^i + Y^i(\hat{z}) = \bar{Z}$. 

Static size-discovery mechanism-design results
Remaining primitives of the dynamic model

- The cumulative inventory shock of trader $i$ is a zero-mean Lévy process $H^i$.

- At time $T \sim \exp(r)$, the asset pays an independent amount with mean $v$.

- Almgren-Chriss holding cost for excess-inventory process $z$ is $\gamma \int_0^T z_t^2 \, dt$.

- Without trade, the total value to trader $i$ is therefore

\[
E \left[ vz^i_T - \gamma \int_0^T (z_t^i)^2 \, dt \right],
\]

where $z^i_t = z^i_0 + H^i_t$. 
The exchange: A dynamic double-auction market

- At a given price $p$, in state $\omega$ at time $t$, trader $i$ demands the asset at some chosen quantity rate $D_t^i(\omega, p) \in \mathbb{R}$.

- For a given price process $\phi$, the total payment by trader $i$ in state $\omega$ is thus

$$
\int_0^T \phi_t(\omega)D_t^i(\omega, \phi_t(\omega)) \, dt.
$$

- The associated excess inventory process of trader $i$ is

$$
z_t^i = z_0^i + \int_0^T D_t^i(\phi_t) \, dt + H_t^i.
$$
Strategic avoidance of price impact


- At price $p$, trader $i$ assumes that each trader $j \neq i$ demands

$$D_t^i(\omega, p) = a + bp + cz_t^i(\omega),$$

for some given $a, b < 0$, and $c$.

- Continuous-time version of the Du-Zhu equilibrium for the demand-function submission game, with price process $\phi$. 
Augmenting with size-discovery sessions

- In equilibrium, the average excess inventory $\bar{Z}_t$ can be inferred from the exchange price $\phi_t = \Phi(\bar{Z}_t)$.

- A size-discovery session at time $t$ generates the cash transfer $T^i(\mu_t, \phi_t)$ and the asset transfer $Y^i(\mu_t, \phi_t)$.

- Size-discovery sessions are held at the event times of a Poisson process $N$ with mean frequency $\lambda$, based on a specific mechanism $(T^i, Y^i)$. Examples:
  - Linear-quadratic mechanism.
  - Walrasian.
  - Conventional dark pool.
Exchange trading with size discovery

- Trader $i$ submits exchange demand process $\mathcal{D}$ and a mechanism report process $\hat{z}^i$, generating the excess inventory process

$$z^i_t = z^i_0 + \int_0^t \mathcal{D}^i(\phi_s) \, ds + \int_0^t Y^i(\mu_s, \phi_s) \, dN_s + H^i_t.$$
Exchange trading with size discovery

- Trader $i$ submits exchange demand process $\mathcal{D}$ and a mechanism report process $\hat{\mathcal{D}}_i$, generating the excess inventory process

$$z_t^i = z_0^i + \int_0^t \mathcal{D}_i^i(\phi_s) \, ds + \int_0^t Y_i^i(\mu_s, \phi_s) \, dN_s + H_t^i.$$

- The stochastic control problem of trader $i$, given other traders’ strategies, is

$$\sup_{\mathcal{D}, \hat{\mathcal{D}}_i} E \left[ z_T^i v - \int_0^T \phi_t^D \mathcal{D}_t \, dt + \int_0^T T_t^i(\mu_t, \phi_t^D) \, dN_t - \gamma \int_0^T (z_t^i)^2 \, dt \right].$$
Exchange trading with size discovery

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$$z^i_t = z^i_0 + \int_0^t \mathcal{D}^i(\phi_s) \, ds + \int_0^t Y^i(\mu_s, \phi_s) \, dN_s + H^i_t.$$ 

- The stochastic control problem of trader $i$, given other traders’ strategies, is

$$\sup_{\mathcal{D}, \hat{z}^i} E \left[ z^i_T v - \int_0^T \phi^D_t \mathcal{D}_t \, dt + \int_0^T T^i(\mu_t, \phi^D_t) \, dN_t - \gamma \int_i^T (z^i_t)^2 \, dt \right].$$

- Equilibrium: market clearing, consistent conjectures, and agent optimality (incentive compatibility and mechanism IR).
Size-discovery reduces allocative efficiency

Proposition

1. Above a stated mean frequency $\bar{\lambda}$ of size-discovery sessions, exchange trading breaks down.

2. For any $\lambda < \bar{\lambda}$, there are 2 linear equilibria. In the more efficient equilibrium, welfare (indeed, every trader’s value) is strictly decreasing in $\lambda$. 
Inventory paths with mechanism reliance on exchange prices
Inventory paths with directly observable aggregate inventory

- Efficient inventory
- Auction with mechanism for agent 1
- Auction with mechanism for agent 2
- Only double auctions for agent 1
- Only double auctions for agent 2

Mechanism occurrences
Policy-related observations

- With competing platform operators, entry of a size-discovery platform is profitable but (in our model) socially harmful.

- As size-discovery sessions become more frequent, exchange volume and depth decline.

- If size discovery is available, traders will use it even though they are better off (in our model) without it.

- Scope for regulation. MiFiD II caps dark pool trading volume.