Fragmenting Financial Markets

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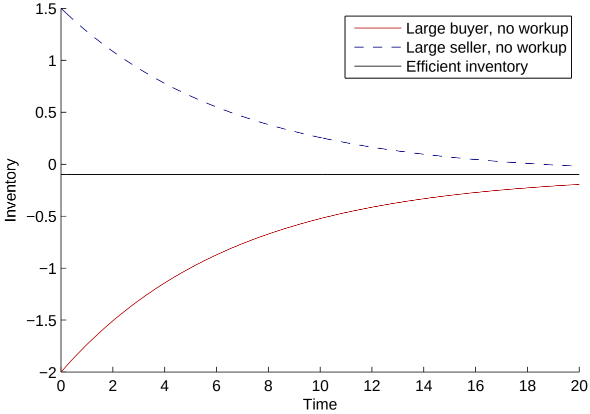
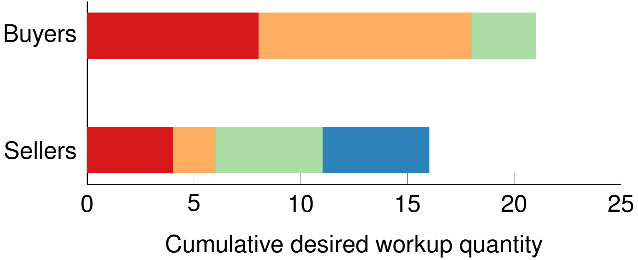
Avoidance of price impact delays efficient trades

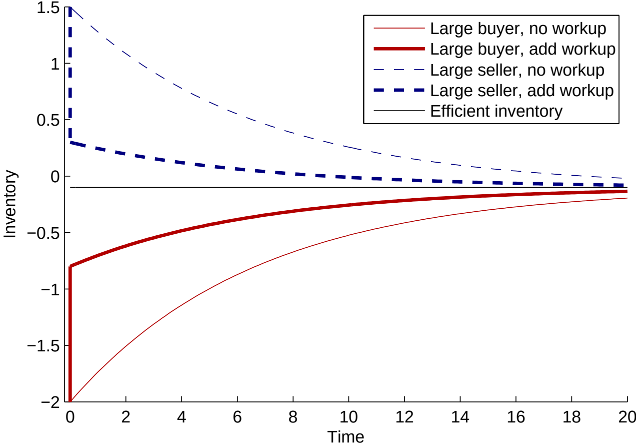


Vayanos (1999), Rostek and Weretka (2015), Du and Zhu (2017), Duffie and Zhu (2017).

A workup session involving 3 buyers and 4 sellers.



Improving the initial allocation with a workup



Source: Duffie and Zhu (2017).

Size discovery in practice

- Treasuries workup trade. Roughly half of inter-dealer trade volume. Boni and Leach (2002), Fleming and Nguyen (2013), Duffie and Zhu (2017).

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- Scope for regulation. In 2018, European MiFiD II

Main findings

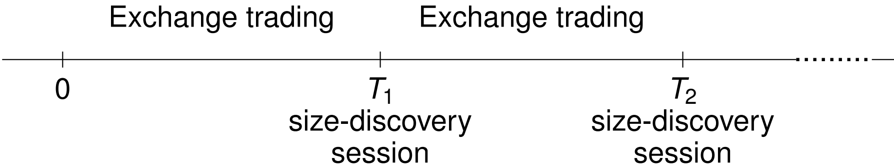
1. Size discovery is highly effective for avoiding price-impact costs and can dramatically improve allocations whenever it is run.

2. The prospect of size discovery, however, reduces exchange market trade volumes and depth.

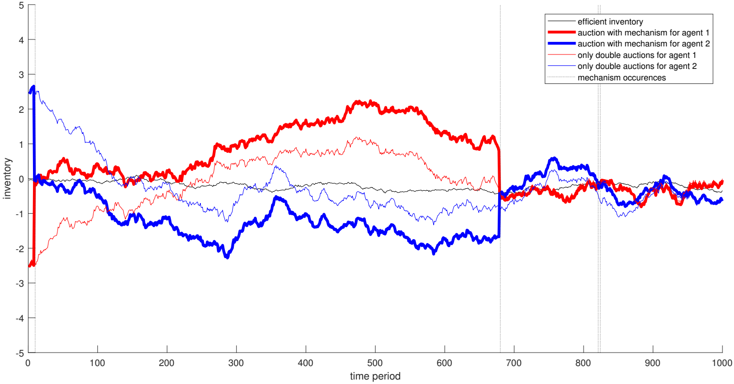
3. The net effect, as modeled, is a reduction in overall allocative efficiency.

4. Ex ante, every investor would strictly prefer the absence of size discovery.

Model Timeline



Inventory paths with and without size-discovery sessions



Static size-discovery primitives

- $n \geq 3$ traders with private initial excess inventories $z_0^1, \ldots, z_0^n$.
- Average excess inventory $\bar{Z} = (z_0^1 + \cdots + z_0^n)/n$.
- Value of excess inventory (later endogenized):

$$V^i(z^i, \bar{Z}) = u^i(\bar{Z}) + \beta(\bar{Z})z^i - K\left(z^i - \bar{Z}\right)^2,$$

  where u^i, β are functions and $K > 0$ is a scalar.
- The unique efficient allocation: $z^i = \bar{Z}$.

Size-discovery session

- Suppose (for now) that $\bar{Z}$ is publicly observable.
- Trader i submits an inventory report $\hat{z}^i$.
- Given reports $\hat{z} = (\hat{z}^1, \ldots, \hat{z}^n)$, trader i gets some cash transfer $T^i(\hat{z}, \bar{Z})$ and some asset transfer $Y^i(\hat{z})$.
- Taking $\hat{z}^{-i}$ as given, trader i solves

$$\sup_{\tilde{z}} \mathbb{E}\left[V^i(z_0^i + Y^i((\tilde{z}, \hat{z}^{-i})), \bar{Z}) + T^i((\tilde{z}, \hat{z}^{-i}), \bar{Z}) \,|\, \mathcal{F}^i\right].$$

A linear-quadratic mechanism for size discovery

- Asset transfer
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- Cash transfer

$$T^i(\hat{z}, \bar{Z}) = \underbrace{\kappa_1(\bar{Z})}_{\text{frozen price}} \hat{z}^i + \kappa_0 \left(n\, \kappa_2(\bar{Z}) + \sum_{j=1}^{n} \hat{z}^j \right)^2$$

$$+ \kappa_1(\bar{Z}) \kappa_2(\bar{Z}) + \frac{\kappa_1^2(\bar{Z})}{4 \kappa_0 n^2},$$

where $\kappa_1(\cdot)$ and $\kappa_2(\cdot)$ are affine and $\kappa_0 < 0$ is a constant.

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where $\kappa_1(\cdot)$ and $\kappa_2(\cdot)$ are affine and $\kappa_0 < 0$ is a constant.

- Budget feasibility: $\sum_i T^i(\hat{z}, \bar{Z}) \leq 0$ for any $\bar{Z}$ and $\hat{z} \in \mathbb{R}^n$.

Static size-discovery mechanism-design results

Proposition: For unique $\kappa_0, \kappa_1, \kappa_2$, this size-discovery mechanism is:

- Strategy proof: Reporting $\hat{z}^i = z_0^i$ is a strictly dominant strategy.
- Ex-post IR: For any $z_0 \in \mathbb{R}^n$ and for the equilibrium strategies $\hat{z}^i = z_0^i$,

$$V^i(z_0^i, \bar{Z}) \leq V^i(z_0^i + Y^i(z_0), \bar{Z}) + T^i(z_0, \bar{Z}).$$

- Efficient: $z_0^i + Y^i(\hat{z}) = \bar{Z}$.

Remaining primitives of the dynamic model

- The cumulative inventory shock of trader i is a zero-mean Lévy process H^i.

- At time $\mathcal{T} \sim exp(r)$, the asset pays an independent amount with mean v.

- Almgren-Chriss holding cost for excess-inventory process z is $\gamma \int_0^{\mathcal{T}} z_t^2 \, dt$.

- Without trade, the total value to trader i is therefore

$$E\left[v z^i_{\mathcal{T}} - \gamma \int_0^{\mathcal{T}} (z^i_t)^2 \, dt \right],$$

where $z^i_t = z^i_0 + H^i_t$.

The exchange: A dynamic double-auction market

- At a given price p, in state ω at time t, trader i demands the asset at some chosen quantity rate $\mathcal{D}_t^i(\omega, p) \in \mathbb{R}$.

- For a given price process ϕ, the total payment by trader i in state ω is thus

$$\int_0^{\mathcal{T}} \phi_t(\omega)\mathcal{D}_t^i(\omega, \phi_t(\omega))\, dt.$$

- The associated excess inventory process of trader i is

$$z_t^i = z_0^i + \int_0^{\mathcal{T}} \mathcal{D}_t^i(\phi_t)\, dt + H_t^i.$$

Strategic avoidance of price impact

- Vayanos (1999), Rostek and Weretka (2015), Du and Zhu (2017).
- At price p, trader i assumes that each trader $j \neq i$ demands

$$\mathcal{D}_t^j(\omega, p) = a + bp + cz_t^j(\omega),$$

  for some given a, $b < 0$, and c.
- Continuous-time version of the Du-Zhu equilibrium for the demand-function submission game, with price process ϕ.

Augmenting with size-discovery sessions

- In equilibrium, the average excess inventory $\bar{Z}_t$ can be inferred from the exchange price $\phi_t = \Phi(\bar{Z}_t)$.
- A size-discovery session at time t generates the cash transfer $T^i(\mu_t, \phi_t)$ and the asset transfer $Y^i(\mu_t, \phi_t)$.
- Size-discovery sessions are held at the event times of a Poisson process N with mean frequency λ, based on a specific mechanism (T^i, Y^i). Examples:
  - Linear-quadratic mechanism.
  - Walrasian.
  - Conventional dark pool.

Exchange trading with size discovery

- Trader i submits exchange demand process $\mathcal{D}$ and a mechanism report process $\hat{z}^i$, generating the excess inventory process

$$z_t^i = z_0^i + \underbrace{\int_0^t \mathcal{D}^i(\phi_s)\,ds}_{\text{exchange trade}} + \underbrace{\int_0^t Y^i(\mu_s, \phi_s)\,dN_s}_{\text{size-discovery trade}} + H_t^i.$$

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- The stochastic control problem of trader i, given other traders' strategies, is

$$\sup_{\mathcal{D}, \hat{z}^i} E\left[z_{\mathcal{T}}^i v - \int_0^{\mathcal{T}} \phi_t^{\mathcal{D}} \mathcal{D}_t\, dt + \int_0^{\mathcal{T}} T^i(\mu_t, \phi_t^{\mathcal{D}})\, dN_t - \gamma \int_i^{\mathcal{T}} (z_t^i)^2\, dt \right].$$

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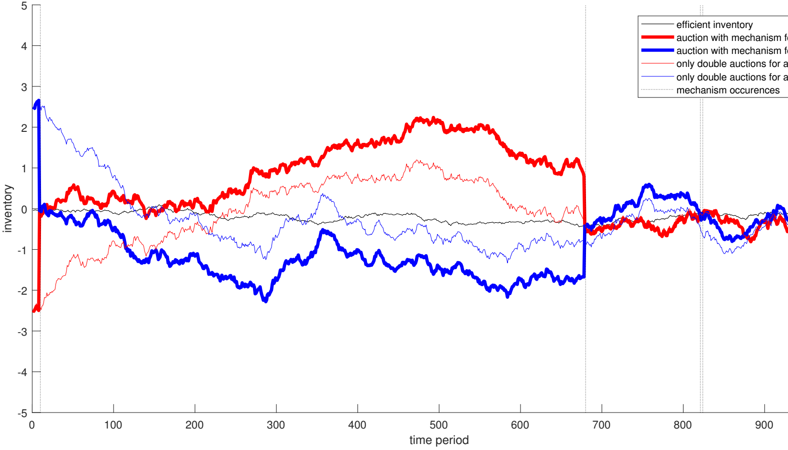
- Equilibrium: market clearing, consistent conjectures, and agent optimality (incentive compatibility and mechanism IR).

Size-discovery reduces allocative efficiency

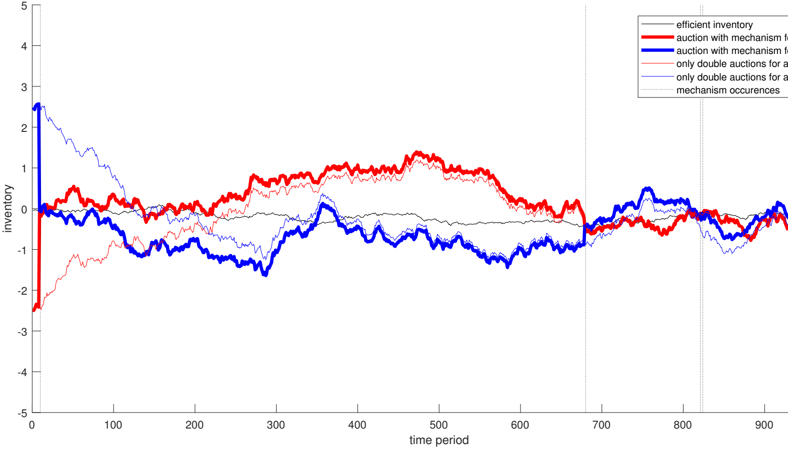
Proposition

1. Above a stated mean frequency $\bar{\lambda}$ of size-discovery sessions, exchange trading breaks down.
2. For any $\lambda < \bar{\lambda}$, there are 2 linear equilibria. In the more efficient equilibrium, welfare (indeed, *every* trader's value) is strictly decreasing in λ.

Inventory paths with mechanism reliance on exchange prices



Inventory paths with directly observable aggregate inventory



Policy-related observations

- With competing platform operators, entry of a size-discovery platform is profitable but (in our model) socially harmful.
- As size-discovery sessions become more frequent, exchange volume and depth decline.
- If size discovery is available, traders will use it even though they are better off (in our model) without it.
- Scope for regulation. MiFiD II caps dark pool trading volume.
- The policy-relevant empirical evidence is limited to equities, and mixed. See: Buti, Rindi, and Werner (2011), DeGryse, De Jong, and Kervel (2015), Nimalendran and Ray (2014), Farley, Kelley, and Puckett (2017).