Continuous-time persuasion by filtering

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Outline

From a survey by E. Kamenica

There are three factors driving behavior and choices in economics:

- preferences;
- technology;
- information.

So, we can (try to) change economic outcomes via:

- incentives, threat of violence, supplying additional goods
- improving technology
- **providing information**.

Applications[1]: financial sector stress tests, grading in schools, employee feedback, law enforcement deployment, voter coalition formation, farmaceutical research, CO_2 emissions reduction.

[1] For references on all of this see Kamenica (2019)

Signaling games: persuasion games

IDEA: a Sender (she) tries to influence for her best interest the action of a Receiver (he), by sending him a message on the state of the world.

Two cases:

- the Sender observes the true state of Nature and she sends a message accordingly: *"cheap talk"*, influencers;
- the Sender designs a mechanism that generates messages as a function of the state that she may not even observe: pharmaceutical companies.

Bayesian Persuasion [Kamenica and Gentzkow (2011)]

The Receiver has a utility function $u(a, \omega)$ that depends on her action a and the state of the world $\omega \in \Omega$. The Sender, the information designer, has utility $v(a, \omega)$ that depends on Receiver's action. They share a common prior μ_0 on Ω.

1. For S finite realization space, Sender chooses a signal $\pi : \Omega \to \Delta(S)$,
2. Receiver observes which signal was chosen,
3. Nature chooses ω according to μ_0,
4. Nature draws s according to $\pi(\omega)$,
5. Receiver observes the realized s,
6. Receiver takes action a.

Why **Bayesian**? The Receiver immediately updates μ_0 to the posterior $\mu_\pi(\omega|s)$ via Bayes.
Assumption: commitment of the Sender on the information policy.

Extensions

Micro:

- Receiver has some private information ✓
- Heterogeneous priors
- Costly signals ✓
- No Sender's committment

Macro:

- Multiple Receivers ✓
- Multiple Senders
- Dynamic environment. ✓

Literature for the dynamic extension:

- **Discrete-time:**
  [Renault et al. (2013), Golosov et al. (2014), Au (2015), Renault et al. (2017), Bizzotto et al. (2021a), Bizzotto and Vigier (2021), Zhao et al.(2024)].
- **Continuous-time:** [Ely (2017), Orlov et al. (2020), Ely and Szydlowski (2019), Bizzotto et al. (2021a), Liao(2021), Ball (2023), Yao (2023), Escudé and Sinander (2023)].

Our dynamic (continuous time) framework

A **linear-quadratic** stochastic **Stackelberg game** with **partial information** and **ergodic criteria**, between a (MF of) Receiver(s) and a Sender.

	State process X	**Message M**	Comment
Sender	Controls: NO	Controls: YES	
Receiver	Observes: NO Controls: YES	Observes: YES Controls: NO	← partial information

Pros	**Cons**
Sender's **commitment is enforced** via the signaling device	The Sender's decision is **static**: no deviation possible once the signaling mechanism is active
Receiver's **rational anticipation**: he gradually gets closer to X	
Allows for several **energy-finance applications**	

Contributions

Mathematical results:

- Problem formulation and solution with key features:
  - linear-quadratic (stochastic) Stackelberg game
  - ergodic criterion
  - asymmetric information (**filtering**) between a (MFG of) receiver(s) and a sender.
- Proof of a **Verification theorem** to solve the (receiver's separated) problem

Economic insights via two examples:

- The informative value of smart meters for electicity consumption reduction $\Leftarrow$ check the paper!
- Firms carbon footprint reduction $\Leftarrow$ today!

General mathematical setting

We focus on a game between a **sender (S)** and a **receiver (R)** of information.

- $(\Omega, \mathcal{F}, \mathbb{P})$ with independent Br. motions W and B of dimension d_W and d_B.
- State variables:

$$\mathrm{d}X_t = (A_X X_t + A_v v_t + a_X)\mathrm{d}t + \mathrm{d}W_t, \quad X_0 = x_0 \in \mathbb{R}^{d_W},$$
$$\mathrm{d}M_t = bX_t\mathrm{d}t + \Sigma \mathrm{d}B_t, \quad M_0 = 0$$

  where $A_X \in \mathcal{S}_{d_W}(\mathbb{R})$, $A_v \in \mathbb{R}^{d_W \times r}$, $a_X \in \mathbb{R}^{d_W}$, $\Sigma \in \mathbb{R}^{d_B \times d_B}$, $b \in \mathbb{R}^{d_B \times d_W}$.
- The receiver's filtration is given by

$$\mathcal{F}_t^R = \sigma(M_s : 0 \leq s \leq t) \vee \mathcal{N}, \quad t \geq 0.$$

- Admissible strategies:

$$\mathcal{A}^R = \Big\{ v = (v_t)_{t\geq 0} \in \mathbb{R}^r, \mathbb{F}^R\text{- adapted} : \mathbb{E}\Big[\int_0^T |v_t|^2 dt\Big] < +\infty, \forall T > 0, \lim_{T\to\infty} \frac{1}{T}\mathbb{E}[|X_T|^2] = 0 \Big\}$$

$$\mathcal{A}^S = \Big\{ (b, \Sigma) \in \mathbb{R}^{d_B \times d_W} \times \mathbb{R}^{d_B \times d_B} \text{ with } \Sigma \text{ invertible} \Big\}$$

- Target: minimizing the *long-time average cost functionals*

$$J^R(v; b, \Sigma) := \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\int_0^T f(X_t, v_t)\mathrm{d}t\right],$$

$$J^S(b, \Sigma; v) := h(b, \Sigma) + \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\int_0^T g(X_t, v_t)\mathrm{d}t\right],$$

where

$$f(x, v) = x^\top F_2 x + F_1^\top x + F_0 + v^\top C_2 v + C_1^\top v, \quad (x, v) \in \mathbb{R}^{d_W + r},$$

with $F_2 \in \mathcal{S}_{d_W}(\mathbb{R})$, $F_1 \in \mathbb{R}^{d_W}$, $F_0 \in \mathbb{R}$,$C_2 \in \mathcal{S}_r(\mathbb{R})$, $C_1 \in \mathbb{R}^r$,
$h : \mathbb{R}^{d_B \times d_W} \times \mathbb{R}^{d_B \times d_B} \to \mathbb{R}$, measurable, g suitable measurable function to be specified in the applications.

Filtering

He has no **direct** access to X and so he uses the **filter of X**:

$$\widehat{X}_t := \mathbb{E}[X_t | \mathcal{F}_t^R], \quad t \geq 0.$$

It is known (see e.g., [Davis (1977)]) that

$$d\widehat{X}_t = (A_X \widehat{X}_t + A_v v_t + a_X)\mathrm{d}t + P_b(t) b^\top dI_t, \quad \widehat{X}_0 = x_0,$$

with (here Σ is the identity matrix)

- The $\mathbb{F}^R$- Br. motion I (**innovation process**), $dI_t = dM_t - b\widehat{X}_t dt, t \geq 0$,
- P_b is the $(d_W \times d_W)$-**error covariance matrix**

$$P_b(t) = \mathbb{E}\left[(X_t - \widehat{X}_t)(X_t - \widehat{X}_t)^\top \middle| \mathcal{F}_t^R \right] = \widehat{X_t^2} - (\widehat{X}_t)^2$$

deterministic, independent of the control, solving the matrix Riccati eq.

$$P_b'(t) = A_X P_b(t) + P_b A_X^\top - P_b(t) b^\top b P_b(t) + \mathbf{1}, \quad P_b(0) = \mathbb{V}[X_0] = \mathbb{V}[x_0] = 0.$$

The **Receiver's problem** is proved to be equivalent to

$$\inf_{v \in \mathcal{A}^R} \widetilde{J}^R(v; b, \Sigma) := \inf_{v \in \mathcal{A}^R} \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}\left[\int_0^T f(\widehat{X}_t, v_t) \mathrm{d}t\right].$$

Q: does the system have a long-run performance? Is the filter stable at infinity?

Exploiting [Wohnam(1968)], under assumptions on A_X and b, there exists a positive definite matrix $P_b(\infty)$, which reads:

$$P_b(\infty) = \lim_{t \to +\infty} P_b(t)$$

and $P_b(\infty)$ is the unique positive semi-definite solution of the Algebraic Riccati Eq.

$$A_{\mathrm{x}} P_b + P_b A_{\mathrm{x}}^\top - P_b b^\top b P_b + \mathbf{1} = 0.$$

Notice that we are dealing with a stochastic control problem with an ergodic criterion but **non-homogeneous state dynamics**!

Verification theorem (general)

On the complete filtered probability space $(\Omega, \mathcal{F}, (\overline{\mathcal{F}}_t)_{t\geq 0}, \mathbb{P})$ supporting an m-dimensional Br. motion $\overline{W}$, consider

$$\inf_{\nu\in\mathcal{U}} J(\nu) := \inf_{\nu\in\mathcal{U}} \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\int_0^T f(\mathbf{X}_t, \nu_t)dt\right],$$

where $\mathbf{X} = \mathbf{X}^\nu$ is an n-dimensional stochastic process solving

$$d\mathbf{X}_t = \mu(\mathbf{X}_t, \nu_t)dt + \sigma(t)d\overline{W}_t, \quad \mathbf{X}_0 = \mathbf{x}_0 \in \mathbb{R}^n,$$

with

- $\mu : \mathbb{R}^n \times \mathbb{R}^d \to \mathbb{R}^n$ measurable and Lipschitz in $\mathbf{x}$ uniformly in ν,
- $\sigma(t)$ is a semi-positive definite deterministic matrix in $L^2(\mathbb{R}_+^{n\times m})$,
- ν belongs to the set of admissible controls $\mathcal{U}$.

Theorem (Verification theorem)

Assume that there exists $\sigma(\infty) \in \mathbb{R}_+^{n\times m}$ *such that*

$$\frac{1}{T}\int_0^T ||\sigma^T(t)\sigma(t) - \sigma^T(\infty)\sigma(\infty)||dt \to 0, \quad T \to +\infty. \tag{1}$$

Moreover, let $V \in C^2(\mathbb{R}^n)$ *with at most quadratic growth and let the Hessian matrix* $H(V)$ *be bounded, and* $\zeta \in \mathbb{R}$ *such that*

$$\inf_{u\in\mathbb{R}^d} \left(\mathcal{L}_\infty^u V(\mathbf{x}) + f(\mathbf{x},u)\right) - \zeta = 0, \quad \mathbf{x} \in \mathbb{R}^n, \tag{2}$$

where

$$\mathcal{L}_\infty^u \varphi(\mathbf{x}) := \mu(\mathbf{x},u)\nabla\varphi(\mathbf{x}) + \frac{1}{2} Tr\left(\sigma^T(\infty)H(\varphi)(\mathbf{x})\sigma(\infty)\right),$$

for all functions $\varphi \in C^2(\mathbb{R}^n)$*. Assume that there exists a measurable function* $\nu^* : \mathbb{R}^n \to \mathbb{R}^d$ *attaining the infimum in* (2)*. Then, if* $\nu_t^* := \nu^*(\mathbf{X}_t)$, $t \geq 0$*, belongs to* $\mathcal{U}$*, it is an optimal control and* $\zeta = J(\nu^*)$*.*

The sender's problem ($\Sigma \equiv Id$)

Once existence of a stationary distribution for the state variables $(X^*, \widehat{X}^*)$ is proved, the *mean ergodic theorem* implies

$$\lim_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\int_0^T g(X_t^*, v^*(\widehat{X}_t^*))dt\right] = \mathbb{E}\left[g(X_\infty^*, v^*(\widehat{X}_\infty^*))\right],$$

so that the Sender's problem is equivalent to

$$\inf_{b\in\mathcal{A}^S} \widetilde{J}^S(b; v^*) := \inf_{b\in\mathcal{A}^S} h(b) + \mathbb{E}\left[g(X_\infty^*, v^*(\widehat{X}_\infty^*))\right] \tag{3}$$

where $(X_\infty^*, \widehat{X}_\infty^*)$ are defined on a suitable probability space s.t., in the topology of weak convergence of measures

$$\mathcal{L}(X_\infty^*, \widehat{X}_\infty^*) = \lim_{t\to\infty} \mathcal{L}(X_t^*, \widehat{X}_t^*), \tag{4}$$

$$\Downarrow$$

The Sender's problem reduces to a **static optimisation problem**, whose objective depends only on the stationary distribution of state variables.

Carbon footprint accounting rules: the (MF)game

Players:

[Regulator/Sender/she] Designs guidelines and constraints for regulation on carbon emissions (direct and indirect: purchased electricity, waste disposal, and business travel, ...) reporting

[Firms/Receivers/he] Decides carbon footprint reduction to a target level and to be below the average carbon footprint of their peers, in a form of best-in-class emulation process.

Reporting rules (regulator)	**Reporting costs** (representative firm)	**Planet** (us)
strict	high	happy
mild	low	unhappy

The state variables

- **Carbon footprint of the representative firm**:

$$dX_t = -\kappa(X_t - \ell)dt + v_t dt + dW_t, \quad X_0 = x_0,$$

- **Information device**: for $(b, \sigma) \in \mathbb{R} \times (0, +\infty)$

$$dM_t = bX_t dt + \sigma dB_t, \quad M_0 = 0,$$

provides information on the firm's true carbon footprint ($b = 0$: norms are so mild that firms have no information).

- $\ell > 0$ stationary level, $\kappa > 0$ speed of mean-reversion;
- v is the firm's control ;
- (b, σ) are the regulator's controls, $\widetilde{b} := \frac{b}{\sigma}$.

The Receiver's problem

[IDEA] Mean-field interaction (ergodic criterion): each player interacts with the long term empirical distribution of the population's state and actions.
The representative firm minimises

$$J^{\mathrm{R}}(v) := \limsup_{T\to+\infty} \frac{1}{T} \int_0^T \mathbb{E}\Big[c(v_t) + \underbrace{\lambda_q (X_t - q)^2}_{\text{best-in-class}} + \underbrace{\lambda_a (X_t - a)^2}_{\text{target } a\neq\ell}\Big] \mathrm{d}t, \quad (5)$$

with $c(v) := \frac{1}{2}\frac{v^2}{\gamma}, \gamma > 0$, the cost of control, $\lambda_a, \lambda_q \geq 0$ and $q \in \mathbb{R}$ satisfies the (mean-field) consistency condition:

$$q = \lim_{T\to+\infty} \mathbb{E}[X_T] - \epsilon\sigma_T,$$

- σ_T is the standard deviation of X_T
- $\epsilon \geq 0$ is the *level of best-in-class target.*

The Sender's problem

The regulator's objective is to minimise the potential damage caused by the firms' carbon footprint:

$$\inf_{(b,\sigma)\in\mathcal{A}^{\mathrm{S}}} \overbrace{h(b,\sigma)}^{\text{cost for the device}} + \limsup_{T\to\infty} \frac{1}{T}\int_0^T \mathbb{E}\Big[\underbrace{c(X_t^*)^2}_{\text{damage's cost}} \Big] \mathrm{d}t,$$

with $c > 0$ and X^* the carbon footprint process at the mean-field equilibrium. This becomes

$$\inf_{(b,\sigma)\in\mathcal{A}^{\mathrm{S}}} h(b,\sigma) + \mathbb{E}\big[c(X_\infty^*)^2\big].$$

The solution - I

- The **representative firm's optimal control** is

$$v_t^* = \kappa(\widehat{X}_t^* - \ell) - \beta\widehat{X}_t^* - \beta\hat{\ell}(q)$$

with $\beta := \sqrt{\kappa^2 + 2\gamma\bar{\lambda}}$ and $\hat{\ell}(q) := \frac{\kappa^2\ell - 2\gamma\bar{q}}{\kappa^2 + 2\gamma\bar{\lambda}}, \bar{\lambda} := \lambda_a + \lambda_q, \bar{q} := \lambda_a a + \lambda_q q$.

- The quantile equilibrium value q_∞^* is

$$q_\infty^* = \frac{\kappa^2\ell - 2\gamma\lambda_a a}{\kappa^2 + 2\gamma(\bar{\lambda} + \lambda_q)} - \epsilon\sigma_\infty(b)\frac{\kappa^2 + 2\gamma\bar{\lambda}}{\kappa^2 + 2\gamma(\bar{\lambda} + \lambda_q)}. \tag{6}$$

- The stationary mean m_∞^* of the process X is $m_\infty^* = \hat{\ell}(q_\infty^*) = q_\infty^* + \epsilon\sigma_\infty(b)$:

$$m_\infty^* = \frac{\kappa^2\ell - 2\gamma\lambda_a a}{\kappa^2 + 2\gamma(\bar{\lambda} + \lambda_q)} + \epsilon\frac{2\gamma\lambda_q}{\kappa^2 + 2\gamma(\bar{\lambda} + \lambda_q)}\sigma_\infty(b) =: m_0 + \epsilon m_1\sigma_\infty(b). \tag{7}$$

The solution - II

- Noticing that $\mathbb{E}[(X^*_\infty)^2] = (m^*_\infty)^2 + \sigma^2_\infty$, the optimisation problem of the Sender is

$$\inf_{z \in (\underline{v}, \bar{\sigma}]} G(z) + \frac{1}{2} c\big[(m_0 + \epsilon m_1 z)^2 + z^2\big],$$

where $z := \sigma^2_\infty(b)$ and where the function G measures the cost to reduce the standard deviation of the stationary law of X.

- A possible form that leads to closed-form expression is

$$G(z) = -\frac{1}{2} \frac{\eta}{\Delta v} \ln \frac{z - \underline{v}}{\Delta v}, \quad z \in (\underline{v}, \bar{\sigma}].$$

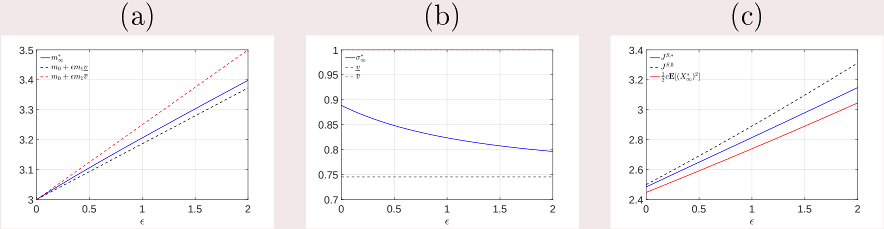

Figure: As a function of ϵ: (a) m_∞^* (b) σ_∞ (c) Damage cost with optimal information provision and without any information provision. Parameters: $\kappa = 0.5$, $\ell = a = 9$, $\gamma = 0.1$, $c = 0.5$, $\lambda_a = 0.25$, $\lambda_q = 0.75$.

- As b increases (more precise device and information), both m_∞^* and $(\sigma_\infty^*)^2$ decrease
- m_∞^* is increasing in ϵ; σ_∞^* decreasing: it is easier to look cleaner in a dirty crowd (greenwashing)!
- Fig. (a): information provision (b high, low $\sigma_\infty^2 \to \underline{v}$) allows m_∞^* to get closer to the lowest possible bound.

To conclude...

- Additional example in the paper (available in ArXiV)
- **Further extensions:**
  - → **two competing** senders
  - → **time-dependent** control for the sender
  - → **beyond linear-quadratic** objective functionals and dynamics
  - → **new applications** (such as the manipulation of environmental score by firms).

Thanks for your attention!

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The Parameters

Consider the following setting:

$$\kappa = 0.5, \qquad \ell = \tilde{\ell} = 5, \qquad u_0 = 100, \qquad \gamma = 1, \qquad h = 0.5,$$
$$p_0 = 50, \qquad p_1 = 100, \qquad g_0 = 0, \qquad g_1 = 500, \qquad \rho = 0,$$

and

$$\sigma = 1 \text{ (fixed)}\ , \qquad X_0 = m_\infty,$$

so that the sender has a **one-dimensional** control.

Return